

# Scale Analysis of the Turbulence Heat Transfer over the Tibetan Plateau with Wavelet Transform

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## 1. Introduction

Turbulent transfer of heat, momentum and trace gases is a phenomenon whose characteristics scale spans over several decades. As a rough sketch, Kolmogoroff's law of 5/3 power well describes observation at length scales between the integral scale and the micro scale, while an established theory is lacked at the scales larger than the integral scale. This region of the scale resides between the spectrum peak of turbulence and that corresponds to daily cycle of meteorological variables, and interfaces the two different scales, i.e. turbulence and larger scale phenomena.

From a practical point of view, "energy closure problem", which currently draws much attention from the micrometeorological community[e.g. 8], necessitates the investigation of this scale region, as the averaging time/length for the Reynolds averaging is usually set within or around this scale region and as its inadequate selection could cause flux underestimation with the eddy correlation technique[e.g. 6].

Purpose of this paper is to identify the scales relevant to turbulence heat transfer by applying the wavelet transform to the data acquired over the Tibetan Plateau, and to discuss possible causes of the energy imbalance.

## 2. Data Analyzed

Data subject to the analysis is a turbulence data acquired with an eddy correlation system over a flat and sparse grassland near Naqu city during GAME-Tibet '98 IOP as a cooperated work between Korean, Chinese, and Japanese scientists. Detailed description of the observation site and the measurement system are given elsewhere[1]. Energy budget study [1] with this data set has already revealed that there is a significant energy imbalance of the order of 20% of the available energy when the turbulent flux are evaluated with eddy correlation technique. From this data set, 9 consecutive data

Table 1: Analyzed runs. Time in BST

Run	Date	start time	end time
1	07/28	10:00:10	17:17:04
2	08/03	10:06:59	17:23:54
3	08/16	10:08:31	17:25:26
4	08/22	10:08:30	17:25:24
5	08/30	09:59:17	17:16:11
6	09/06	10:00:50	17:17:44
7	09/12	08:00:38	15:17:32
8	09/13	10:08:31	17:25:25
9	09/14	10:06:51	17:23:45

were selected for the subsequent analysis (Table 1).

## 3. Wavelet Transform

Wavelet transform came into the micrometeorological community in the late '80s [e.g., 3] and quickly became a common tool with an anticipation that it could better describe the turbulence eddy than the Fourier transform do. Mathematical basis of the wavelet transform and its application to the turbulence are fully described elsewhere[e.g., 2; 5] and will be briefly explained here. Wavelet transform of the time series  $f(t)$  can be written as,

$$W_f(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(t) \cdot \psi\left(\frac{t-b}{a}\right) dt \quad (1)$$

where  $\psi(t)$  is called mother wavelet, which usually have finite support unlike its Fourier counterpart, i.e. trigonometric functions. Parameters  $a$  and  $b$  are called "scale" and "translate" respectively. As shown in equation (1), wavelet transform decomposes original time series into a wavelet packet specified by the scale  $a$  and the location  $b$ .

Multiplication of the wavelet transform of two time series  $f(t)$  and  $g(t)$ , i.e.,  $W_f(a, b)W_g(a, b)$  indicates the component of the covariance between  $f(t)$  and  $g(t)$  with the scale  $a$  observed at the time location  $b$ . Integration of this over the time, i.e.,

$$E_{fg}(a) = \int_{-\infty}^{\infty} W_f(a, b)W_g(a, b)db \quad (2)$$

is called wavelet covariance, and means the covariance component with the scale  $a$ . This wavelet covariance, often called wavelet cospectrum, is the wavelet counterpart of Fourier cospectrum.

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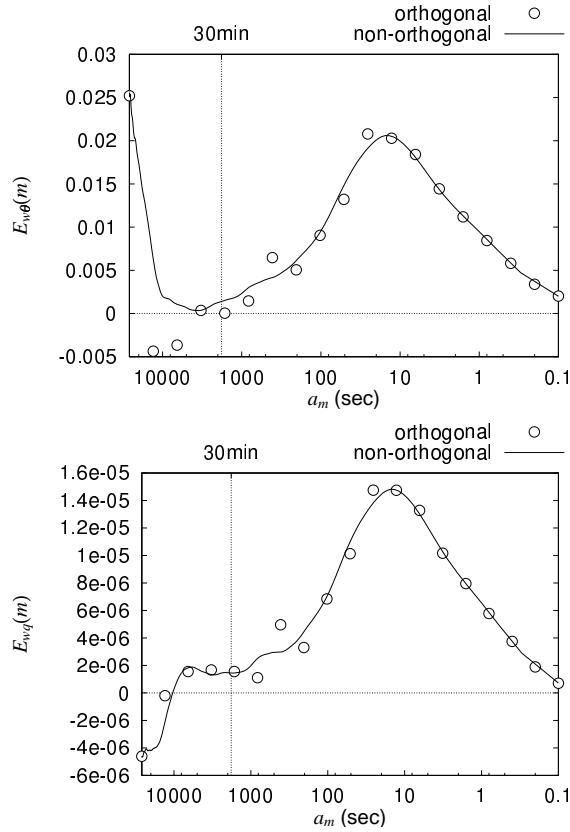


Figure 1: Calculated wavelet covariance for run 6. Upper panel: sensible heat flux, lower: latent heat flux

In this study, wavelet covariance for the heat transfer,  $E_{w\theta}, E_{wq}$  is calculated for each of the Runs listed above. Haar wavelet is selected as a mother wavelet due to its simplicity and an computational methodology described in [4] was used to implement the orthogonal wavelet transform.

#### 4. Results and discussion

Figure 1 shows the calculated wavelet covariance of the sensible and latent heat flux for Run 6. The length scale corresponds to the 30 minutes averaging time that is used to evaluate Reynolds average

Table 2: Contribution of the heat transfer at the scale larger than 30 minutes relative to the total heat transfer(in %).

Run	$w-\theta$	$w-q$
1	1.4	0.7
2	-1.8	2.3
3	11.6	-5.5
4	-9.2	4.1
5	-13.0	3.4
6	3.4	1.3
7	-17.0	11.7
8	-3.9	3.4
9	-6.1	2.4

is also indicated in the figure. It is apparent that while the most of the heat is transferred at the scale of the order of 1 to 100 seconds, some portion of the transfer occurred by the eddy larger than the size corresponds to the 30 minutes; this suggests that the averaging time of 30 minutes is not long enough to capture all of the heat transfer. It should be also noted, however, that the large amount of the sensible heat is also transferred at the scale around 7 hours where the negative value of the latent heat flux is observed. This behavior of the wavelet covariance at the larger scale was found to differ with each Run: some Runs have heat transfer with opposite direction to those shown in Figure 1 within the scale range beyond 30 minutes.

In order to identify the contribution of the heat transfer at the larger scale, calculated covariance are integrated from 30 minutes to the total length of the data and for whole range of the scale, and the ratio of the two are calculated. The results are list in Table 2. These numbers indicates that the contribution of the eddy larger than 30 minutes is not negligibly small, but not large enough to explain all of the energy imbalance found in the surface energy budget.

When the large scale components around 7 hrs were reconstructed into a time scale using inverse wavelet transform, it is identified as a trend in the temperature and humidity with time scale of one day. However, lack of the data longer than one day in the data set prohibited the further investigation.

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