

# Composite Variability of the Surface Fluxes and its Effect on the Turbulence Statistics in the Unstable Atmospheric Surface Layer

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## 1 Introduction

Monin-Obukhov similarity (MOS) theory (see e.g. Haugen, 1973) has been established in the past half century as a way to describe the turbulence field within the steady and horizontally uniform atmospheric surface layer (ASL). It successfully predicts the mean, variance and higher moments of velocity components and scalars, and the universal functional form for the dimensionless moments has been empirically determined in numerous experiments (see e.g. Kader and Yaglom, 1990, for reviews). For a non-uniform ASL, which can be generally found over most natural landsurface, however, it is still not known whether MOS can be extended to such conditions.

Recent studies (e.g. Brutsaert and Sugita, 1990; Parlange and Brutsaert, 1993, and references cited therein) have revealed that, over complex terrain, mean quantities such as  $\bar{u}$ ,  $\bar{\theta}$  and  $\bar{q}$  follow MOS to some extent above a height called the “blending height”, of the order of  $50z_0$ , where  $z_0$  is the surface roughness, up to the height of the ASL. The variance, however, appears to be more sensitive to the variability of the surface and the situation appears to be more complicated. Indeed, some recent studies (e.g. Katul *et al.*, 1995; Asanuma and Brutsaert, 1996) reported that the variance can be altered considerably by the surface variability.

It is the purpose of this paper to shed some light on this problem of non-uniform surface and to quantify the effect of the surface variability on the turbulence moments by using a simple conceptual analysis for the unstable turbulence field with a one-dimensional composite-type distribution of the surface fluxes.

## 2 Conceptual Analysis

### 2.1 Homogeneous Surface

First, it is hypothesized that, over a homogeneous surface in a certain range of  $\zeta \equiv z/L$  under unstable condition, the MOS relationships for the local free convection (Tennekes, 1970; Wyngaard *et al.*, 1971), or equivalently, the dynamic-convective and convective sub-layer scheme of directional similarity (Kader and Yaglom, 1990), can be applied to the mean quantities and variances of  $\theta$ ,  $q$  and  $w$ , where  $L \equiv -u_*^3/(k\beta w'\theta'_0)$  is Obukhov's

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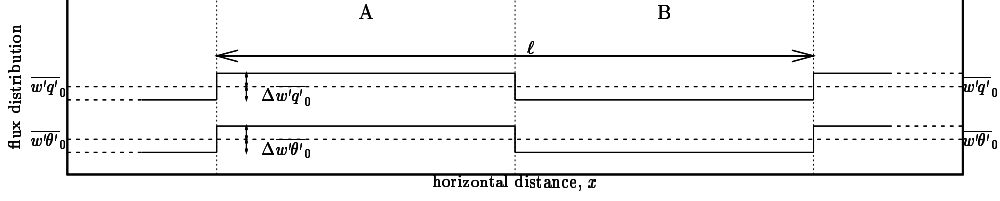


Figure 1: Hypothesized flux distribution of  $\overline{w'\theta'}_0$  and  $\overline{w'q'}_0$ .

stability length. They can be written, for the mean quantities, as

$$\frac{\partial \bar{c}^\circ}{\partial z} = A_c \frac{c_*}{z} (-\zeta)^{-\frac{1}{3}} \quad (1)$$

and for the variances, as

$$\frac{\sigma_w^\circ}{u_*} = C_w (-\zeta)^{\frac{1}{3}} \quad \frac{\sigma_c^\circ}{c_*} = C_c (-\zeta)^{-\frac{1}{3}} \quad (2)$$

where  $c$  can be  $\theta$  and  $q$ ,  $c_* \equiv \overline{w'c'}_0/u_*$  the scaling variable for the scalar  $c$ ,  $\sigma$  the standard deviation, and  $^\circ$  is used to indicate the quantities evaluated over the homogeneous surface.

## 2.2 Composite-type Variability

Next, the surface is assumed to exhibit a one-dimensional composite-type variability in the surface fluxes,  $\overline{w'\theta'}_0$  and  $\overline{w'q'}_0$  (Figure 1). It is further assumed that there is no mean wind and that the turbulence in each column is in equilibrium with the surface below without lateral interaction so that (1) and (2) can be applied for each column with the surface flux values of the surface underneath. This condition requires that the variability of the fluxes be small compared to their absolute value, i.e.  $\chi_c \equiv \Delta \overline{w'c'}_0 / \overline{w'c'}_0 \ll 1$ .

By applying (1) and (2) for each column, the turbulence statistics for each column can be calculated, and it is possible to obtain a “regional” average of the turbulence characteristics by taking the average for whole terrain. By defining  $\xi$  as the ratio of the values of these statistics above the non-uniform terrain to the one above the uniform terrain, the effect of the surface variability on these statistics can be quantitatively evaluated by expressing the  $\xi$ ’s in terms of  $\chi_\theta$  and  $\chi_q$ . For the mean quantities, it can be readily shown that

$$\xi_H \equiv \frac{\widetilde{\partial \bar{\theta}}}{\partial z} \bigg/ \frac{\partial \bar{\theta}^\circ}{\partial z} = \frac{1}{2} \left[ (1 + \chi_\theta)^{\frac{2}{3}} + (1 - \chi_\theta)^{\frac{2}{3}} \right] \simeq 1 - \frac{1}{9} \chi_\theta^2 \quad (3)$$

$$\xi_E \equiv \frac{\widetilde{\partial \bar{q}}}{\partial z} \bigg/ \frac{\partial \bar{q}^\circ}{\partial z} = \frac{1}{2} \left[ (1 + \chi_q)(1 + \chi_\theta)^{-\frac{1}{3}} + (1 - \chi_q)(1 - \chi_\theta)^{-\frac{1}{3}} \right] \simeq 1 + \frac{2}{9} \chi_\theta^2 - \frac{1}{3} \chi_q \chi_\theta \quad (4)$$

where  $\sim$  denotes the values evaluated as the regional average over the non-uniform surface. Similarly, for the variances, with the definition  $\xi_a \equiv \widetilde{\sigma_a} / \sigma_a^\circ$  where  $a$  can be  $w$ ,  $\theta$  and  $q$ , one obtains

$$\xi_w \simeq 1 - \frac{1}{18} \chi_\theta^2 \quad (5)$$

$$\xi_\theta \simeq 1 + \frac{1}{9} \chi_\theta^2 \quad (6)$$

$$\xi_q \simeq 1 + \frac{1}{2} \chi_q^2 + \frac{5}{18} \chi_\theta^2 - \frac{2}{3} \chi_q \chi_\theta \quad (7)$$

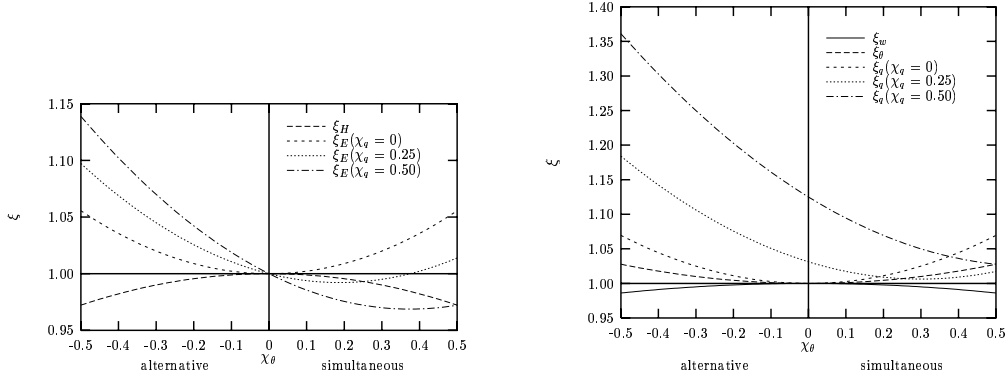


Figure 2: Dimensionless heterogeneous functions,  $\xi$ 's, for the dimensionless gradient of mean quantities (left) and for the variances (right).

Equations (3) through (7) are plotted in Figures 2. Note that as the sign of  $\xi_\theta$  and of  $\xi_q$  are taken in the same manner, the larger value of  $\overline{w'\theta'_0}$  and  $\overline{w'q'_0}$  occur simultaneously when  $\xi_\theta$  and  $\xi_q$  have the same signs, and that the phase of the flux variability is opposite when  $\xi_\theta$  and  $\xi_q$  differ in their sign.

### 3 Discussion

The results shown in Figure 2 reveal some significant features of the turbulence over non-uniform terrain, and they are discussed below.

(1) The surface flux variability results in the modification of dimensionless turbulent statistics due to the nonlinear dependence of these statistics on the surface fluxes. In other words, MOS, in its strict sense, cannot be applied to a non-uniform surface. For the range of the relative variability of the fluxes studied here, however, this effect on the mean quantities is almost negligible, while the variances are relatively sensitive to it. (2) The different roles of an active and a passive scalar in the turbulence field result in different responses to the surface variability. This immediately suggests a break-down of the similarity between these two scalars, i.e. temperature and humidity, under certain conditions of surface variability, and, simultaneously, casts a shadow on the use of Bowen ratio method over non-uniform terrain. But, in practice, the effect of the flux variability on the Bowen ratio should normally be fairly small except in extreme cases. (3) Passive scalar statistics are influenced not only by the magnitude of the variability of the surface fluxes, but also by the phase difference between the distributions of the temperature and the humidity flux. Especially, the latter effect is most distinct when these fluxes are distributed alternatively.

These results are not inconsistent with earlier findings from field experiments. For instance, Brutsaert and Sugita (1992) observed a marked dissimilarity between  $\overline{\theta}$  and  $\overline{q}$  under conditions of surface variability associated with strong drying of the soil moisture. For the variances, Asanuma and Brutsaert (1996) found that over a non-uniform surface the normalized variances of the specific humidity were enhanced and those of  $\theta$  were affected only slightly while those of  $w$  were left unchanged.

## 4 Conclusion

By using a simplified conceptual analysis with similarity theory, it was demonstrated how the variability of the surface fluxes can alter the turbulence field. The effect of this variability on the moments of the vertical wind speed and the temperature was shown to be expressed as,

$$\xi = \xi(\chi_\theta) \quad (8)$$

while that on the moments of a passive scalar,  $q$ , was expressed as,

$$\xi = \xi(\chi_\theta, \chi_q, \alpha_{\theta q}) \quad (9)$$

where  $\alpha_{\theta q}$  refers to the characteristic phase difference between the flux distribution of  $\overline{w'\theta'}$  and  $\overline{w'q'}$ .

In the analysis, idealized conditions were considered by adopting some simplifying assumptions, such as the neglect of the surface shear and horizontal entrainment between vertical columns. To account for these phenomenon,  $\zeta$  and  $\ell/L$  should be probably also among the list of the relevant parameters for  $\xi$ 's, where  $\ell$  is the characteristics length of the flux distribution (Figure 1).

The ensemble averaging used here to derive the regional values of the normalized moments is associated with spatial averaging, as obtained from measurements by aircraft, and measurements at a fixed point, such as those with a micrometeorological tower, would not quite achieve the results presented here. This illustrates the implications of different types of measurement techniques under conditions of surface variability, and also the questionable applicability of Taylor's frozen turbulence hypothesis under such conditions.

The methodology employed here to quantify the effect of the surface flux variability may be useful in providing some guidelines and direction for the future study of land-atmosphere interactions over complex terrain.

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