Runoff runon process in the light of queuing theory.

Runoff genesis, runoff connectivity and upscaling.

Marie Alice Harel - Emmanuel Mouche

emmanuel.mouche@cea.fr

Climate and Environment Sciences Laboratory

91191 Gif sur Yvette, France

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LSCE

I'm very glad to give this talk this afternoon here in your neighbor city of Tsukuba, and I'm really very grateful to Dr. Onda [ph] to having invited me to give this talk and Jérémy Patin, my former Ph.D. student in France. I'm going to talk about runoff generation on plots. Here are slopes with catchments with a special focus on the runoff runon process that they will present and the connectivity with the runoff on these slopes.

This work is part of a work done by Marie Alice Harel who is a Ph.D. student working with me. In fact, it's just in the continuation of what Jérémy Patin did with me a few years ago during his doctoral on the study of runoff prediction under simulated or natural rainfall.

In different countries in collaboration with the IRD which is The French Institute for Research and Development. So, the experiments have been performed mainly in the Houay Pano in Lao PDR in the North of Laos and in Thies in Senegal.

Runoff production under simulated or natural rainfall

from 1 m^2 to 10^4 m^2

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Houay Pano – Lao PDR

Thies - Sénégal

- Ribolzi O., Thiebaux J.P., Bourdon E., Briquet J.P., Chaplot V., Huon S., Marchand P., Mouche E., Pierret A., Robain H., De Rouw A., Sengtahevanghoung O., Soulileuth B., Valentin C, Lao Journal of Agriculture and Forestry 2008
- Ribolzi, O., Patin, J., Bresson, L.M., Latsachack, K.O., Mouche, E., Sengtahevanghoung, O., Silvera, N., Thiébaux, J.P., Valentin, C., Geomorphology, 2010
- Mügler C., Planchon O., Patin J., Weill S., Bariac T., Mouche E., Journal of Hydrology, 2011
- Patin J., Mouche E., Ribolzi O., Valentin C., Latsachack K.O. Journal of Hydrology, 2012
- Ribolzi O., Mouche E., Valentin C., Latsachack K.O. Journal of Hydrology (submitted)

Here you can recognize. This is the catchments where we are working with the people of the IRD. It’s absolutely similar to the catchment that I visited this morning and I'm sorry I cannot pronounce the name of the catchment.

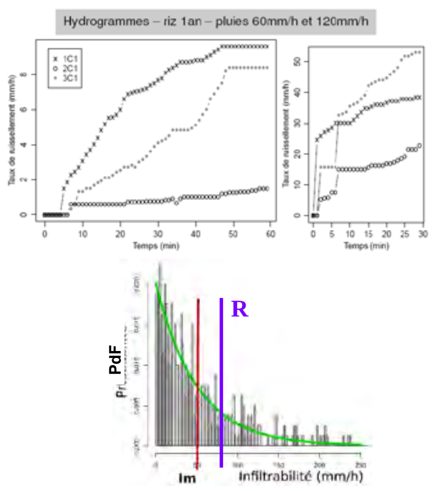
So, in these catchments, there are many experiments and the streamflow is monitored with streamflow discharge stations that surround. There is some rainfall usually and there are some experiments performed on plots under different types of landcovers either on their natural rainfall or simulated rainfall. This catchment is monitored since more than 10 years by the IRD.

Here’s another experiment located in Thies in Senegal, which aims at understanding the runoff formation and the wheel formation. Under simulated rainfall, you can see here the rainfall device and are plot here. It’s a 4 x 10 meter plot. We’ve been working on these different experiments, and here are the scientific predictions on the runoff production.

Runoff - 1 m^2 plots

Infiltration heterogeneities

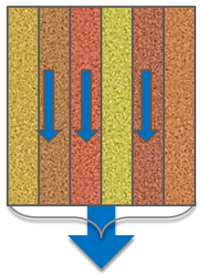
Thesis J. Patin, 2011 - INPL Nancy



(Hawkins 1982)

Plot or hillslope

I_1 I_2 I_3 I_4 …



Parallel strips model

$$I(R) = I_m(1 - exp(-R/I_m))$$

(Yu 1998)

I will just remind a few results of Jérémy's thesis. Jérémy worked partly on 1 square meter plots under either natural or simulated rainfall. One of the results of that – most of you know because you observe the same thing at your catchment as I saw this morning - is that the infiltration rate is very heterogeneous. For instance, here you have got the hydrogram of three experiments performed under simulated rainfall on 1 square meter plots, which have the slope, the same pedology and so on. As you can see, the hydrogram depends on the plots though it's the same for – it means that the infiltration rates varies luckily from one meter to one meter and moreover when you increase the rainfall rates, the infiltration rate increases also and that is not predicted by Horton's law. [Unclear] There is Horton's law which states that whatever the rainfall rates, the infiltration rates withstand to the constant value which is the soil hydraulic conductivity.

To explain this kind of compartments, we use the model of Hawkins in 1982, who supposed that a plot is heterogeneous and maybe viewed as a model with parallel strips, each one having its own infiltrability. That means that you have 1 square meter plots with all these strips with infiltrabilities which is I1, I3 which are distributed let's say exponentially here and if you adopt this model, what do you say? You say that all those strips which have an infiltrability below the rainfall will produce here because this population will produce runoff and all those strips which have an infiltrability greater than the rainfall will infiltrate on the rainfall.

So, you understand that when the rainfall is increasing, the population producing runoff will increase. This is the type of model which explained quite very well the compartment observed here which is that first, the infiltration rate varies from one place back to another one and second, the infiltration rate, the asymptotic infiltration rate increased with the rainfall. Assuming an exponential law, what you get is an infiltration rate which depends on the rainfall R though this relationship which is rather simple.

The thing interesting in the exponential law is that it depends only on one parameter which is I max which is the maximum infiltration rate that you get when the rainfall rate is very large. That is while all the strips, all the infiltrabilities are solicited and all the strips produce runoff.

This model was in fact used by Yu in 1998 in Australia to interpret clouds under simulated and natural rainfall and later used also by Van Dyke [ph] in Indonesia.

Runoff - 1 m^2 plots

I_m distribution

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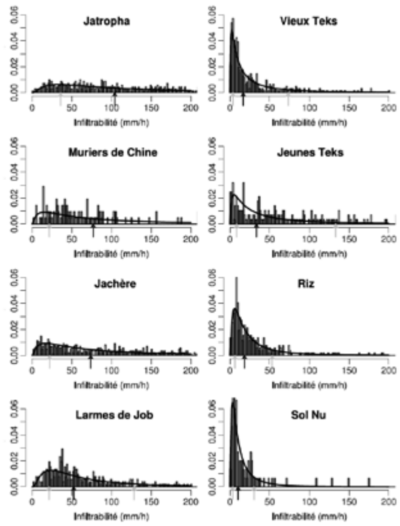
(J. Patin Thesis, 2011)
(Patin 2012)

I_m is log-normally distributed and depends on landcover



Jérémy showed that when you use this model and you try to interpret the distribution of the maximum infiltration, you prove that the maximum infiltration is distributed log normally and the parameters of the log number are low and depends on the landcover. All these points here are experiments performance plots for different type of landcover. That is old peaks, young peaks, rise, bare soils, [Unclear], I don’t remember the name in English and Jatropha.

Runoff on a heterogeneous soil surface

1D parallel strips model works well but is too crude.

- Microtopography ?
- Infiltrability distribution in 2D ?

(Woolhiser 1996)

This 1D parallel strips model works very well. It's very simple because it depends only on one parameter but it's too crude. A few years ago, we wondered how to take into account microtopography and the fact that that the infiltrability is in fact distributed in 2D whatever the scale of observation, the plot, the hillslopes, or the catchment.

Runoff on a heterogeneous soil surface

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1D parallel strips model works well but is too crude.

- Microtopography ?
- 2D Infiltrability distribution

Simplified infiltration model (Hawkins, Yu, …):

$R > I$: runoff (no ponding)
$R < I$: no runoff

We decided to investigate the 2D infiltrability distribution issue assuming an infiltration or rather a simplified infiltration model, which is the one that was used by Hawkins and Yu in their parallel strips model and which is if the rainfall is superior to the infiltration rates, there is runoff. If it's inferior, there is no runoff. During this, in fact, we neglect ponding. We assume that we have instantaneous ponding, or if you want, if we work under simulated rainfall, we assume that we waited for a sufficiently long time to observe the asymptotic infiltration rates.

Runoff on a heterogeneous soil surface

Bibliography

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Work of Wood (1986), Corradini (1998), Nahar (2004), … aim to define equivalent infiltration rate at the scale of a hillslope.

We are interested by what happens at the pixel scale and what is the traduction at a larger scale (Mueller 2007, Gomi 2008, Sen 2010).

- runoff generation,
- dynamics of runoff patterns,
- connectivity,
- effect of soil infiltrability distribution.

What do we know about runoff generation on 2D heterogeneous soil surface? There has been the work of Wood, Corradini, Nahar but all these authors wanted to define equivalent infiltration rates at the scale of the plot or a hillslope or the catchment, and what we were interested in was how - the question or the issues we wanted to tackle was how do the runoff, how runoff is generated at the pixel scale and normally what is the traduction at larger scale, and in doing that, we followed the papers of Mueller 2007, Gomi 2008, and the recent paper of Sen in 2010.

The issues we wanted to address were more specifically what is the runoff generation mechanisms at the pixel scale, how the runoff patterns organize and eventually connect together, what is the dynamic of runoff patterns, how do they connect and how to introduce a measure of connectivity and finally, what is the effect of the soil infiltrability distribution on this runoff dynamics?

Runoff on a random infiltrability soil surface

Runoff-runon, coalescence, and connectivity

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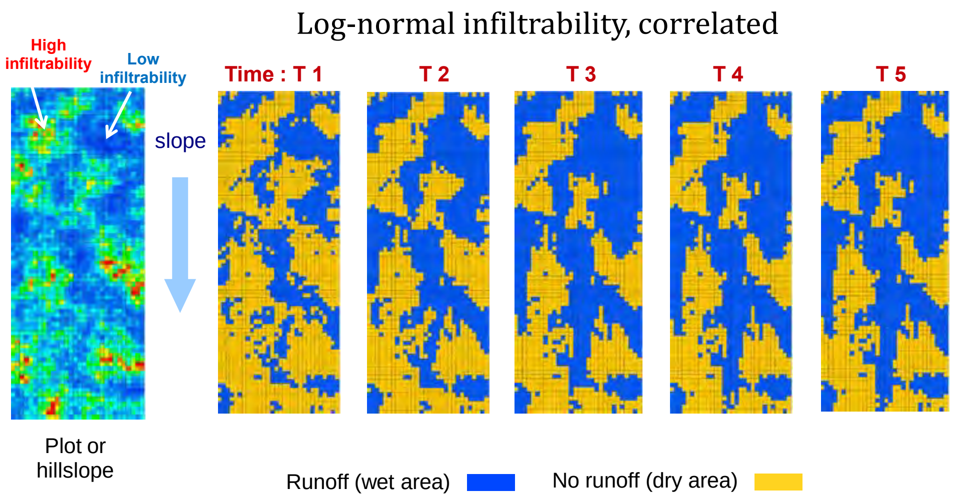


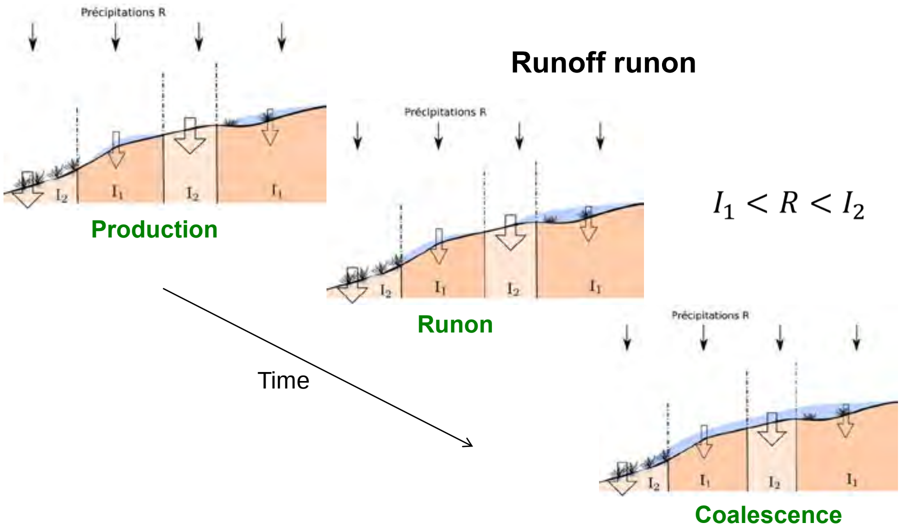
To introduce the concepts, I will show you some results of numerical experiments, which is - let us consider first 2D infiltrability distribution on the soil here. This is a lot normal infiltrability correlated in space. Here are the zones of high infiltrability in red and the zones of low infiltrability in blue. They are to describe appropriately their transparency. When the rain starts at time T1, all of the zones which have a low infiltrability produce runoff. This is what you observe here. In blue, blue means that the area is wet, runoff is generated, and yellow means that this area is dry, there is no runoff. This area corresponds with a high infiltrability which means that all the rainfall infiltrates here, but here due to the low infiltrability, there's an excess rainfall and the rainfall minus the infiltrability produce runoff flowrates in this region.

What happens? This flowrate is generated here. It's going downslope and flowing to pixels which were initially dry. There's some runoff products, there's some runon, and depending of the infiltrability rate value here, there will be some runoff produced finally as a result of the runoff-runon process.

Progressively as time evolves, what you see is that runoff produced here is going to downwards infiltrating all the dry areas here and establish some connection between the different initially wet patches. Here's one disconnected from this one, this one is disconnected from this one, and all these patches, white patches are disconnected from this one, and due to the runoff-runon process, these patches connect and finally, it connects the top of the slope with the bottom of the slope here. You see you have a connected pathway going from up to down. Here, in doing that, I introduce the concept of runoff-runon and the concept of connectivity.

Runoff-runon

(Corradini 1998, …)



$I_1 < R < I_2$

Let’s come back to runoff-runon. What is runoff-runon? Let’s consider a slope with three pixels of a different infiltrability, one pixel with a high infiltrability framed by two low infiltrability pixels. I mean low such that the rainfall is greater than the infiltrability of this pixel, as it is greater than this infiltrability, runoff is produced. This runoff organized, is flowing down infiltrates in I2 but here we assume that the rainfall is lower than I2, but if you’re doing the sum of the rainfall plus the runoff fluid coming from pixel one if you do the sum, you may have a total flowrate or an effective rainfall rate arriving on pixel two which may be superior to I2. This is the case here. Part of this flowrate is infiltrating and the rest is flowing down and connects with the I1, another pixel with I1 infiltration rate. This explains you how the runoff-runon process works and how to initially disconnect two disconnected runoff patterns maybe connected here to form a single pattern. This is what we call coalescence of runoff patterns. This was the runoff-runon process.

Connectivity

(Gomi 2008, ...)

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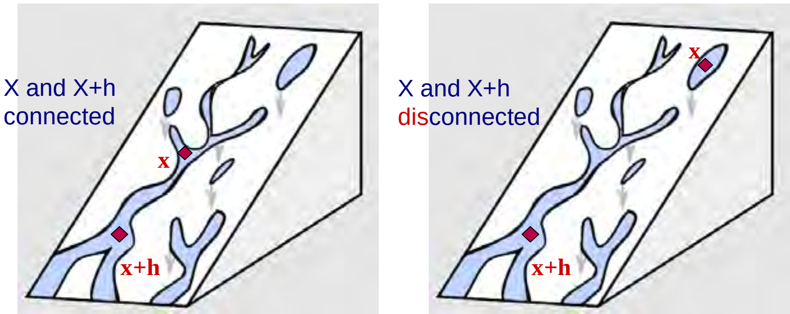
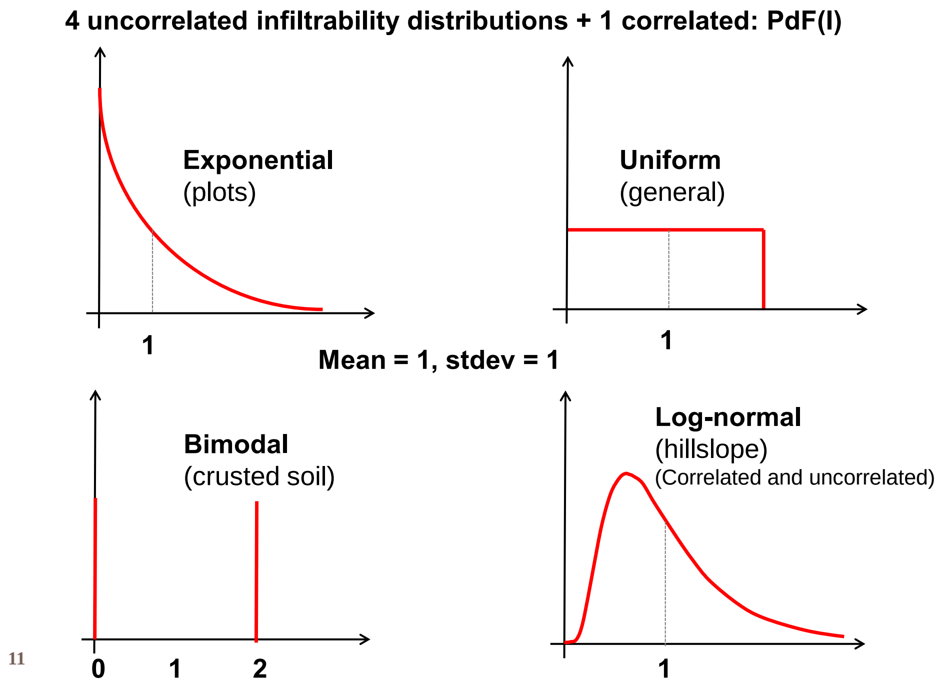


Connectivity function
(Allard 1993)

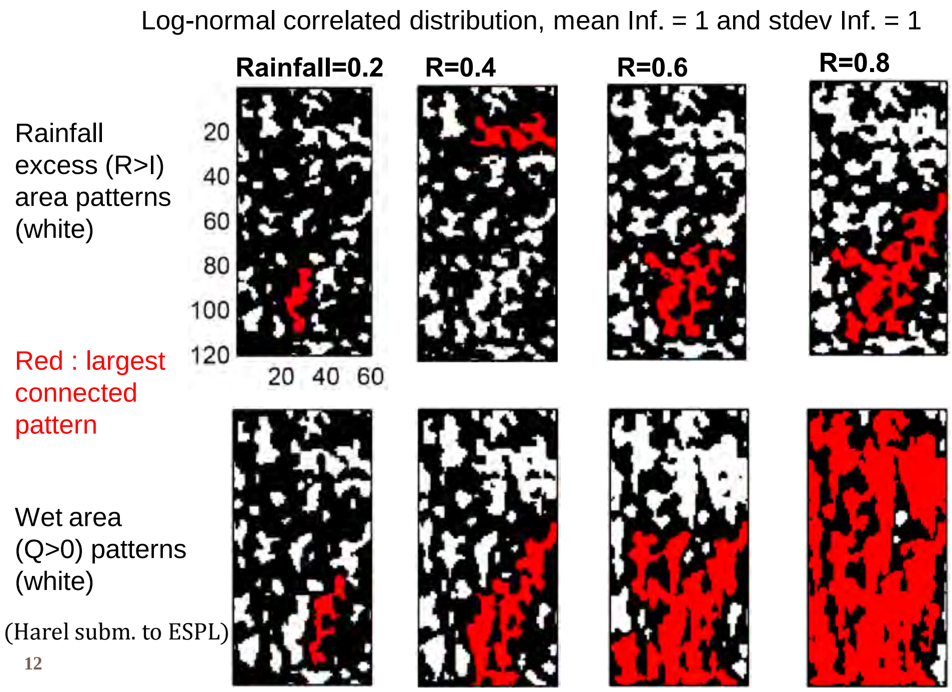
$$T(h) = Prob(x \ and \ x + h \ connected)$$

What about connectivity? The connectivity was discussed by different authors and among them, there was a very good paper, they did a research of 2008 concerning connectivity, the catchment that I visited this morning. Well, you know what is connectivity. You know what means two points connected. It means that if you consider two points X and X plus H in a runoff pattern, we will say that they are connected. If there is a wet pathway going from X to X plus H and they are disconnected, if there is no continuous wet pathway from this point to that point, here you have to pass over this dry area. There are different ways to quantify the concept of connectivity. The one that we adopted is the one proposed by Allard in Geostatistics in 1993, which is just simply the probability - he proposed to define the connectivity function as a function of the line distance between the two points H which the probability that X and X plus H are connected.



I introduced the concepts. What we wanted to do in the first step was to understand how runoff-runon process works with time for transient rainfall, for constant rainfall, how do the runoff, the wet areas - do they connect, so how organized is the runoff patterns and how can they produce, I would say, flow on the streams and so on. For this, we considered four types and we wanted to understand - excuse me, I forgot to say what is maybe the most important thing - how these dynamics depend on the soil infiltrability properties. We considered four uncorrelated infiltrability distributions plus one correlated, which are the exponential that you saw in the model of Hawkins and Yu. Here is the PdF of the infiltrability. This type of distribution applies generally at the plot scale. We wanted to look at the bimodal infiltrability distribution which fits for the crusted soil. Do you know what is a crusted soil? The place where you got some crust. There is no water infiltrating. The place where is bare soils, no crust, water is infiltrating. You can model it through a bimodal infiltrability function. Zero for the place where there's some crust and two for the place where there are no crusts.

All this distribution has the same mean and the same standard deviation which is 1. The last distribution is the low normal which is the one used by Corradini, Nahar, Govindaraju and which applies to the hillslope scale and we considered a correlated case and an uncorrelated. What we mean by uncorrelated is uncorrelated in space. That is white noise. All these signals are white noise distributed according to these distributions.



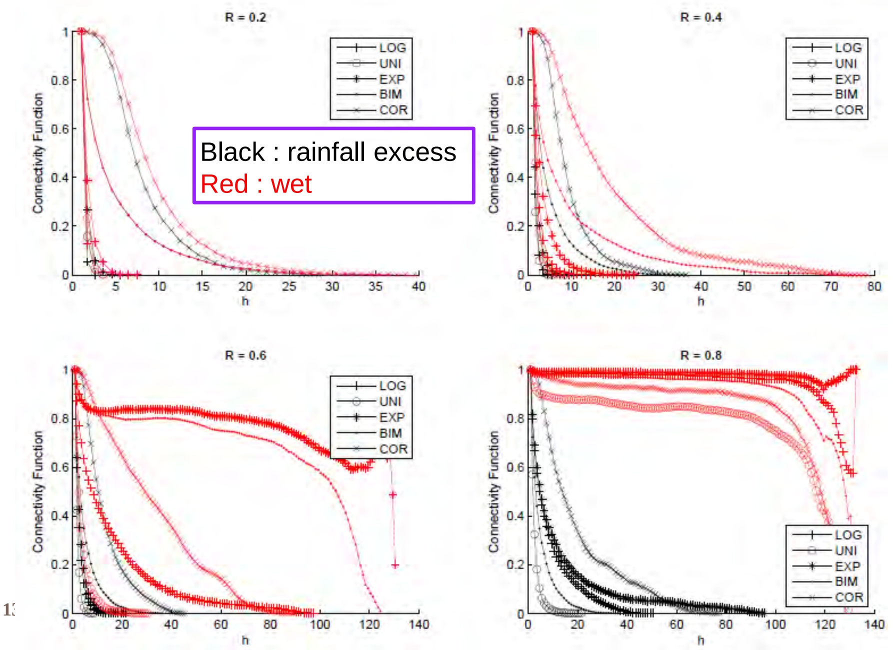
Let me show you the results of these first numerical experiments. These are the results obtained with the log-normal correlated infiltrability. What you have here at the top, these four figures represent the rainfall excess area patterns in whites. That is here's initially you produce an infiltrability distribution, then you construct a binary image in white and in black. In white, you present the patterns which have an infiltrability lower than the rainfall, which will produce runoff, and in black, these are the patterns which have an infiltrability greater than the rainfall that will produce any runoff.

When you increase the rainfall, the value of the rainfall evidently, the area of these patterns increase. This is what you observed here to here to here to here and then they may connect to each other. There's no physics here, it's just mathematics. What I represented in red is the largest connected pattern in each figure, the largest connected pattern. Now, the four figures at the bottom of the slide represent the response of the system when you apply the rainfall. These are the response in terms of runoff but more exactly, here again, we produce a binary image which is if it's wet, if there's some runoff, the pattern is in white, if there's no runoff, the pattern is in black.

What do you see? You see that when the rainfall increases, the wet patterns connect and become greater and greater, and above all what you see is that here in red again, we have the largest pattern. The size of the largest pattern increases and invades all the domain. That means that all the domain is connected or almost all the domain is connected. I wish I have been clear.

Now, there's another thing that you must have in mind that is when the rainfall is very low - when the rainfall is very low, the runoff rate which is produced is quite low. The runoff excess areas much resemble to the wet area zones, and when the rainfall rate tends to 0, the two figures must be the same. I wish I'm clear but this is something which is rather fundamental, but when the rainfall increases, the difference between the two fields increase due to the runoff-runon process. You see here for example, this pattern which produced runoff. It gives a wet pattern which becomes connected to the downstream patterns, thanks to runoff-runon process and you can see here the patterns which are increasing, the wet area patterns which are increasing as the rainfall is increasing. At the end, everything is connected.

Connectivity function (omnidirectionnal)



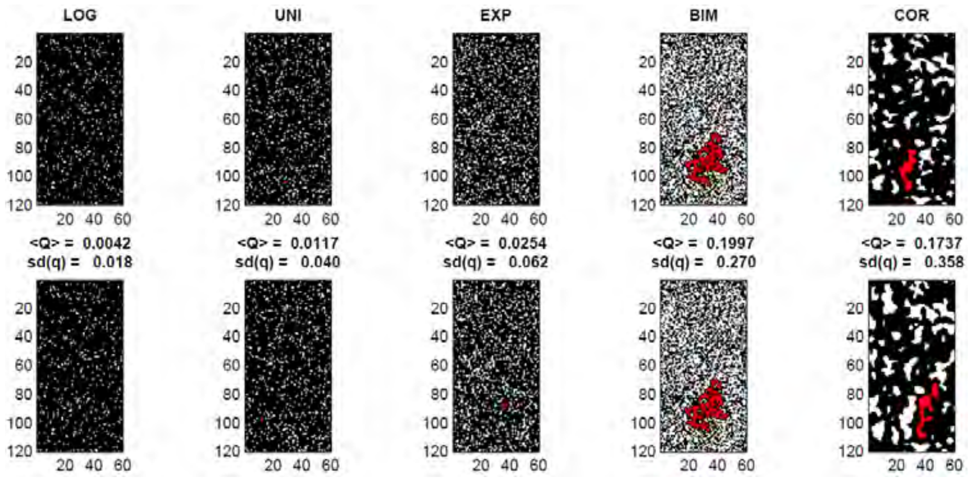
Now, what you can do is take the connectivity function of Allard and plot this connectivity function for these four values of rainfall. The connectivity function, what does this connectivity function say? It says that when the line distance is zero, that means that one point is always connected with itself so the probability is 1. This is what you have here.

When the line distance is very small, you may expect that the two points are always connected, but as soon as the line distance increases, the connection, the probability of connection decreases drastically, especially when the rainfall is low. When the rainfall is increasing, this is what you observe here. You saw that all the runoff patterns connecting to each other and at the end producing a single connected pattern between the top and the bottom of the slope and this is what you see here. You see that the connectivity function is 1 whatever the line distance up to a line distance of 100, 120.

Another thing which must be observed on this figure is - I'm sorry, here you have different infiltrability distributions, so I plotted all the infiltrability distribution. What you have to consider is the symbol COR. That means log-normal correlated. That is the cross. It is this one here. In fact, you have two curves. You have a black one and a red one. The black one is the connectivity function of the rainfall excess area. That is the connectivity of this white pattern here. All these white patterns are the excess, the rainfall excess zones or patterns or areas as you will.

Here in black, I represented the connectivity of these white patterns and in red the connectivity of these wet patterns. When the rainfall is low, connectivity is nearly the same, but when the rainfall increased, the connectivity of the rainfall excess area increased a little bit, thanks to what I told you just a few minutes ago. You see the area or the connection between the different rainfall excess area is increasing. This is what you see here, the black curve here and here but due to runoff-runon process, the connectivity between the wet areas is increasing much more rapidly as you can see here. At the end, everything is connected, all the patterns are connected, all the wet patterns are connected.

R=0.2



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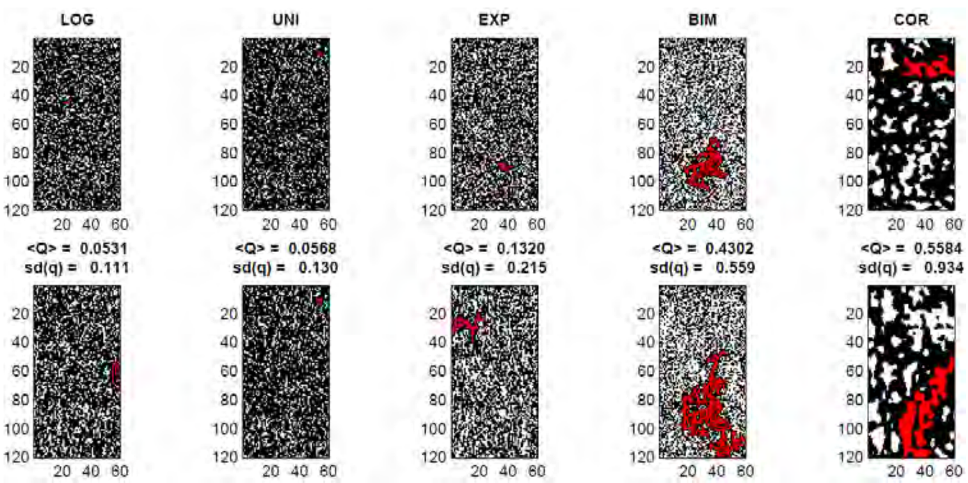
Let’s do the same with the five distribution, the four uncorrelated, the log-normal, uncorrelated, uniform, exponential bimodal and here is again the correlated. That is the one that I just showed you. This is another realization. No, no, it is the same realization I'm sorry.

What do you see? Follow rainfall? Well, the correlated is producing a quite large area, but the bimodal, the crusted soil produced a large connected pattern here but the other distribution produced very, very small pattern. I can see them here or just for the exponential. The same for the wet areas.

Here is the one and here is the other one.

R=0.4



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Now, let’s increase the rainfall. You see, more and more wet patterns are connected for the bimodal, let’s say for exponential but connection for the log-normal and uniform is very low.

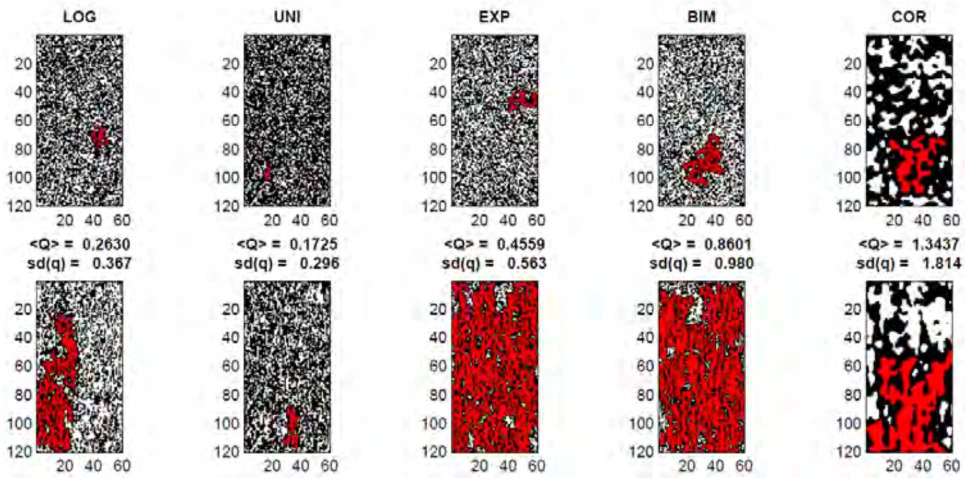
＜この後の、R=0.6、0.8 の説明の後で、再び以下の R=0.4 に関する説明がなされた。＞

Just a little bit of physics concerning the bimodal. Why do we connect strongly with the bimodal, the bimodal distribution? Just simply because when the infiltrability is 0, you produce the maximum runoff flowrate that is the rainfall. Infiltrability is equal to 0 so the runoff flowrate flowing from one pixel to another pixel is just for the rainfall rates.

This is not the case for the uniform because for the uniform when there is a rainfall, the runoff flowrate produced is the rainfall minus the infiltration rates. Moreover, there’s a high probability that this small runoff flowrate may infiltrate in the downward pixel because the distribution is uniform. All the infiltrabilities have the same probability. That is not the case for the bimodal.

I won't go further on on this probabilistic argument, but I hope that you see what I mean.

R=0.6



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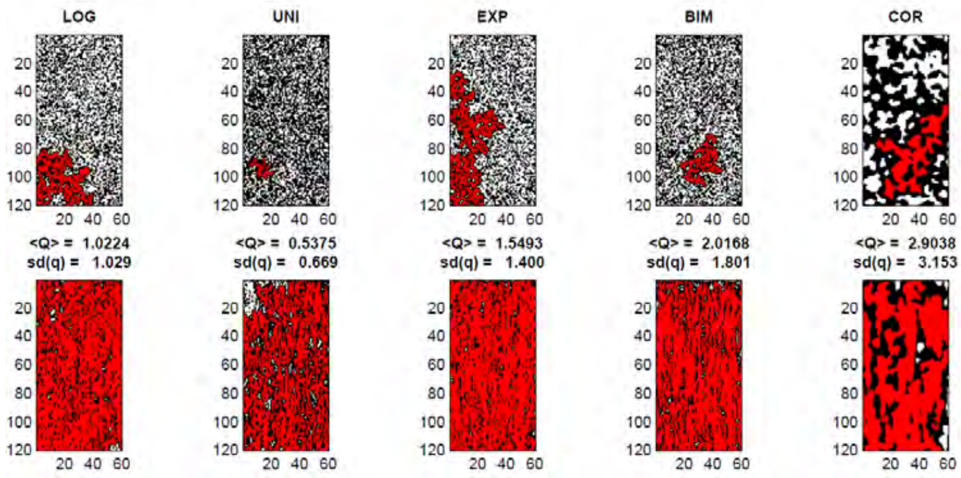
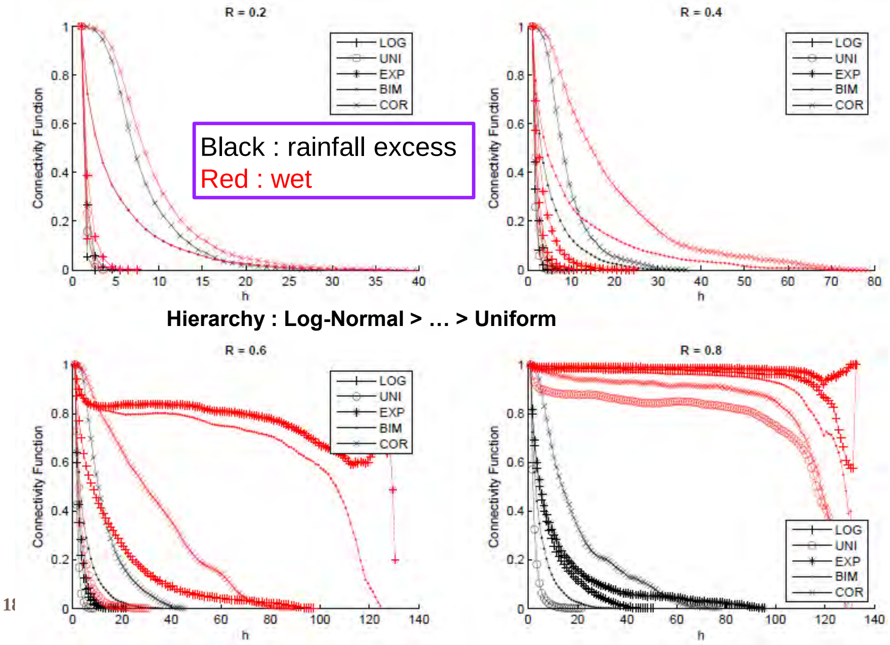
I'm not going to comment on that. You see now that for the bimodal and for the exponential too, almost all the patterns are connected and the top of the bottom is about to be connected,

R=0.8



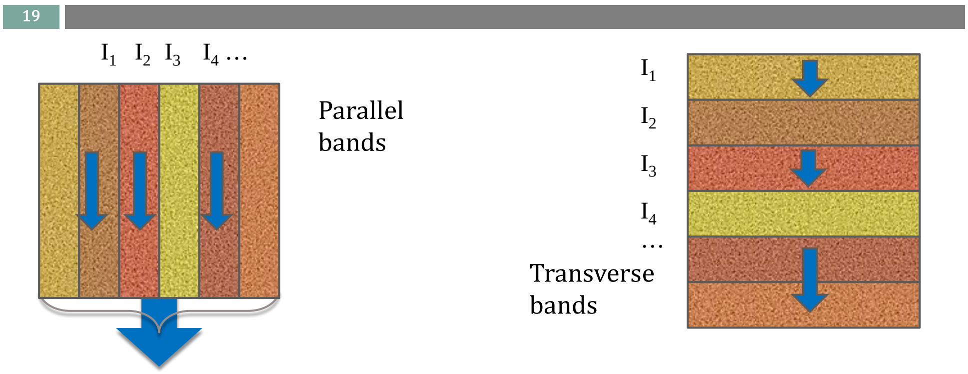
and this is the case of a rainfall which is equal to 0.8.

Connectivity function (omnidirectionnal = unidirectionnal)



This is what we find when we consider now all the other distributions together. In fact, we found that the hierarchy in terms of connectivity is the log–normal, is the more connected and the least connected is the uniform distribution. This is very interesting, very nice but we have no theoretical framework to explain that. These are just numerical experiments. This gives you some idea. You can explain with your hands as we say in France, explain with your hands what happens but no theoretical framework and it's rather complicated. Overall, what I show you here is our experiments, numerical experiments performed under constant rainfall. The thing is more and more complicated when you consider a transient rainfall.

1D transverse band model



Issues :

- Runoff production at the outlet : effective Q(R) ? (flowrate)
- Influence of infiltrability distribution on runoff ?
- Connectivity ?
- 2D = 1D parallel + 1D transverse ? (eg. Effective Perm.)

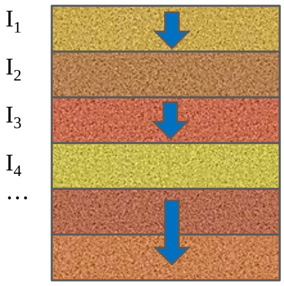
So, we decided to step back and considered only 1D problem but that's not the parallel bands models, which has been already used by Hawkins 1987, but we proposed to consider the 1D transverse band which is the only model in which you can have runoff-runon process. Indeed here, if you think about it, in this type of model, there's no runoff-runon process. The runoff produced by one band because its infiltrability is lower than the rainfall or the runoff arise at the bottom of the domain and does not flow over pixels of different infiltrability. This is a linear model. The runoff that you produce is just the sum of the runoff produced by each band which has an infiltrability lower than the rainfall. This is not the case here. If this band is producing runoff, runoff is produced and then the runoff is going from this band to this side of the band and remember the linear sketch that I showed you, you may have - all the water may reinfiltrate, so water here may reinfiltrate here, water here may reinfiltrate here, water here may flow over this pixel and reinfiltrate in another pixel and so on. This is the only type of 1D model, which may explain runoff which holds the runoff-runon concept.

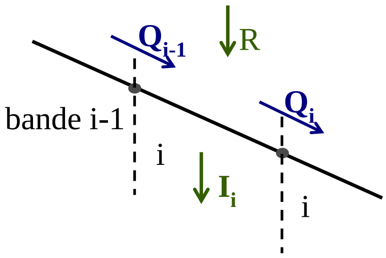
So, the issues that we wanted to tackle with this 1D traverse band model is how can we explain the runoff production at the outlet? Is there an effective law for the runoff flowrate for this system which depends on the rainfall? What is the influence of the infiltrability distribution? What is the connectivity? Finally, could we make some kind of approximation where the connectivity or - I don't know - the flowrates in 2D, the examples that I just showed you may be expressed as the added attributes sum of the results of the 1D parallel model and the 1D transverse, like for instance in the effective permeability definition. In 2D, there's a maximum [Unclear] which states that the permeability is lying between the [Unclear] and the arithmetic permeability. I don't know if what I'm saying sounds to you but in hydrogeology, hydrology, this is something which is very classical.

Theoretical approach

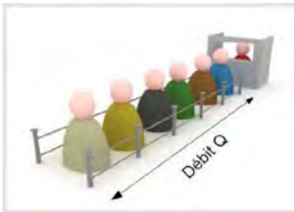
queuing theory (constant rainfall)

Transverse model





$$Q_i = Max\{Q_{i-1} + R - I_i, 0\}$$



$$W_i = Max\{W_{i-1} + S_i - \tau_{i-1}, 0\}$$

waiting – service - interarrival

(Lindley 1952, Jones 2009)

Let's write just an equation which describes the type of dynamics. Again, here we consider only a constant rainfall case. You have three pixels. This is a vertical cut, the pixel I minus 1, I – I forgot, I plus 1. The runoff flowrates flowing outwards of the pixel I is just equal to the rainfall plus the runoff flowrate coming from the upward pixel, Q minus I minus 1 but minus that's just the mass balance the equation, the infiltration rates in pixel I. This may be lower than 0. If this is the case, it means that all the water is infiltrating, so the runoff flowrate coming out from pixel I is 0. You have to choose between the maximum of this mass balance equation and 0 in each pixel. This is exactly the same equation as the one that you find in queuing theory. Do you know what is the queuing theory? It's the branch of mathematics which was developed in the early 50s to explain how a queue appears at the counter and now it has invaded all the mathematics of network technology because we use it in Internet, in the ATM distribution and so on and so on.

How do I establish the parallel Q which is the runoff flowrate? Just let me describe how does it work here, the Q. You have the counter which delivers something during a time which is the service time which delivers the service which service time S. The people in the queue wait. Let's say the people here, the guy in yellow here waits for a certain time which will be the waiting time, that will be index I because he's the customer I and the last parameter of the problem is the interarrival times between two customers.

This equation describes that the time that the customer I in the queue will have to wait as a function of the time it takes, of the service time furnished to the customer I, the interarrival time between customer I and I minus 1.

I was not the guy who observed the equivalence between the two laws. In fact, it was Jones, Australian mathematician who said that hey, these are the same equations so we could use all the queuing staff, I mean all the queuing results developed for this theory since 40 years for runoff prediction on plots in 1D under constant rainfall. So, he wrote it in a small paper and since no more news. When with Marie Alice we read the paper, we said, hey, that's a good idea. We should prospect in this way. In fact, we continue to prospect and we found that the way was a marvelous way.

Queues

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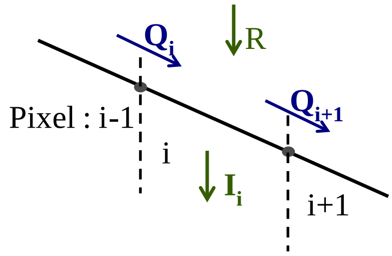
(Thesis M.A. Harel)
(Harel submitted to WRR)



Distribution function $F(Q \leq x)$

Integral equation
Lindley (1951)

$$F(x) = \int_0^\infty F(y)\underbrace{g(x-y)dy}_{\text{Convolution of R \& I Pdfs}}$$

Link between I (and R) and Q statistics

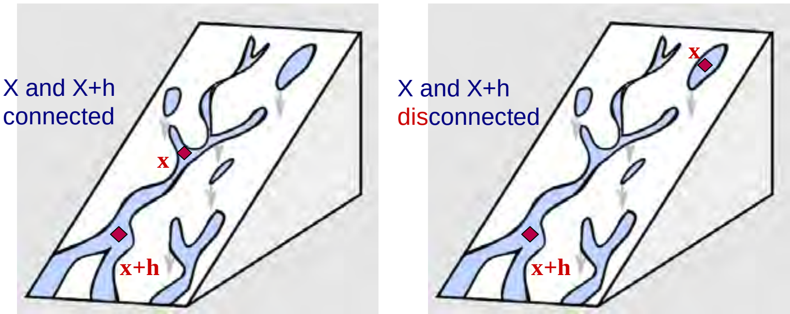
This is the one I'm going to just show you in the next transparencies.

The queuing process relies on the single – the fundamental function is the distribution, the function of the waiting time. Here, it's the runoff rate. I will talk only about hydrological variables, not queuing variables since now.

The fundamental function is the probability - that's the distribution function which is the probability that the runoff is less than a certain value. This function, this distribution function obeys to the Lindley integral equation here. So F is this function, the distribution function of the runoff array and G is the convolution products of the PdFs of the rainfall because you may assume that the rainfall is distributed at random in each pixel. Many authors considered this hypothesis like Wood or Corradini, Monte Carlo simulation, so the convolution between the rainfall PdF and the infiltration PdF. Here, in this relationship, you have the link, this relationship expresses the link between the infiltration PdF and the Q PdF, the statistics of I and the statistics of Q.

Connectivity function

Connectivity function (Allard 1993)



$$T(h) = Prob(x \ and \ x + h \ connected)$$

This is the first mathematical tool. The second one that we used to study the 1D transverse model is the connectivity function that I just introduced to you in the 2D experiments, which is the probability that two points are connected.

5 distributions I : 4 uncorrelated + 1 correlated

Permanent rainfall

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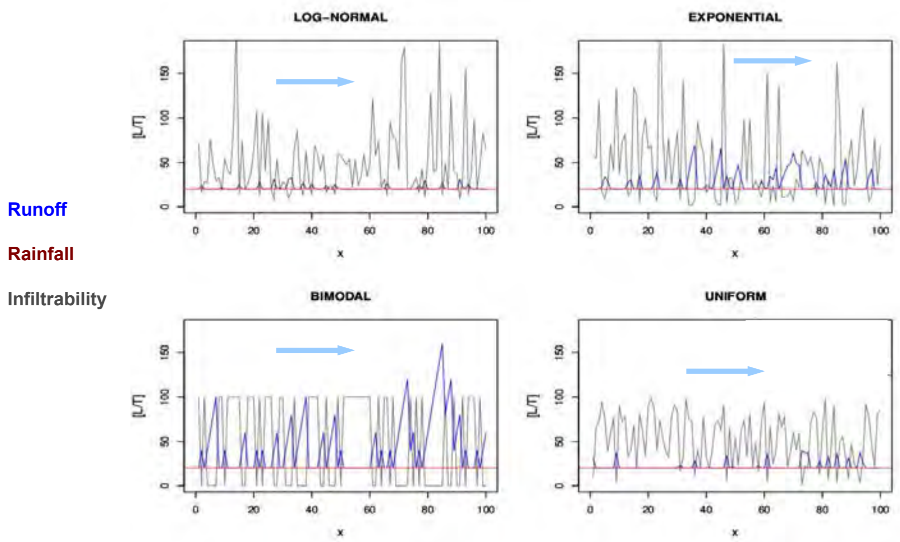
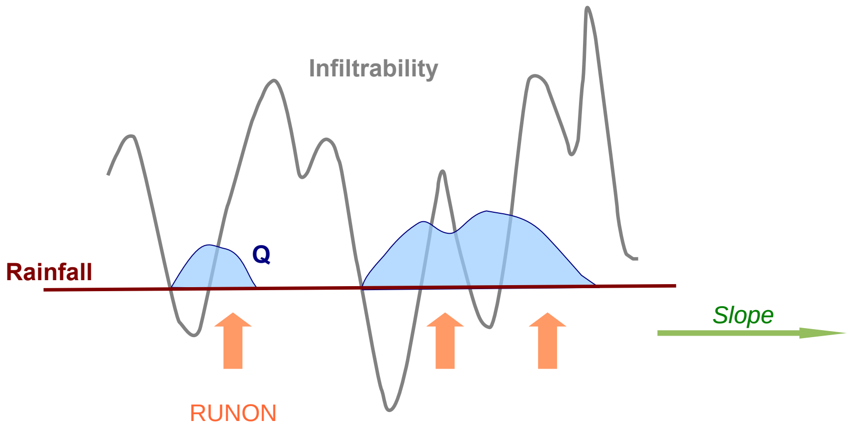


So, let me show you just figures of runoff prediction in 1D. Here in gray you have the infiltrability distribution for the four considered distribution, log-normal, exponential, uniform, and so on. Here's in gray. This is the bimodal, you recognize. It's 100, that is 2 or 0 and the rainfall level, here it's 25 and the runoff flowrate in blue here. What you see, you see the runoff-runon process for instance here. I'm coming back.

Runoff-runon



In areas where the rainfall is greater than the infiltrability, some rainfall. This is what I explained just at the beginning of my talk. You produce some runoff. Then you come to a pixel with a higher infiltrability so you consider the rainfall plus the flowrate coming from this pixel which infiltrates progressively that you come to a dry area because the rainfall here is much lower than the infiltrability. You start again with some runoff prediction, which is a little bit infiltrated because here's a high infiltrability peak, then a low infiltrability peak. You can produce some flowrate and down. Then the flowrate is infiltrating progressively in this zone of high infiltrability. So, that's runoff prediction, runon. I should have begun with my talk with that maybe, maybe that's more understandable that the initial figure that I showed you. I'm sorry.

Fundamental result of queuing theory

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In an infinite domain submersion occurs for R = <I>

$$\rho = \frac{R}{<I>}$$

Fundamental result of the queuing theory is that in an infinite domain submission occurs when the rainfall is equal to the mean infiltration rates. That's a universal law which does not depend on the infiltration distribution. It may be correlated or uncorrelated. That's fundamental, and in terms of queuing process, it says that if the service time is longer than the mean arrival time, the queue grows infinitely, but is the service time at the counter is relative. The service time is much lower than the interarrival time between two customers. The queue tends to wear a stationary state.

Many people working on runoff prediction either at the field or numerically on their computer observed that this law is verified but no one has proved it mathematically. That's just the results of their queuing theory and all the standard works performed in queuing theory assume low cuts value of flow less than 1.

A few results (ρ<1)

Exponential : Erlang 1904 - Bimodal : a few results based on Gravey 1989

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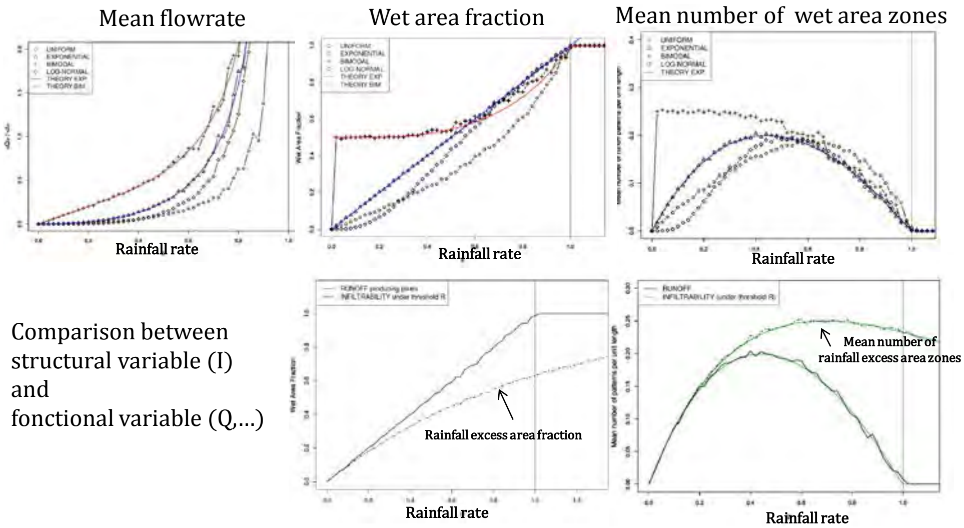

We did some experiments with the different infiltration distributions and here are a bunch of results. Before, I have to say that when we considered all these distributions, uniform, exponential, and so on, we went through the literature and we wondered which were the authors who tackled the different infiltrability distribution that we wanted to study, and in fact, we just found one single distribution that is the exponential who was already studied by Erlang, a Danish mathematician in 1904, so at the beginning of the century, the last century and that's all. Nothing about the uniform, nothing about the bimodal, nothing about the log-normal. Mathematicians considered binary distributions and I do not remember - polynomials, time exponential and distribution and so on but none of the hydrologic distribution that we want to consider.

We tried to develop some original results. That's what we did for the bimodal and we proposed a few results based on the paper of Gravey in 1989. That's all. For the other distributions, we have no theoretical framework, but our aim was to understand with the hands, how this distribution works.

Let me show you first the mean flowrate. Here's the mean flowrates in the domain as a function of the rainfall rates. What you observe is the divergence of the mean flowrates, when you hold the rainfall rates tend to what, that's what I showed you. That's the queue stationary for a row less than 1, with a hierarchy which is the highest flowrates occurs for the bimodal and the lowest for the uniform. Again, what we observed in 2D. The wet area fraction, that's the percentage of the domain which is wet recovered by runoff. One minus the wet area fraction is the dry area fraction.

Here's the line predicted by Erlang theory which states that the wet area fraction for an exponential infiltrability distribution increases linearly with the rainfall rates. We have a very straight compartment for the bimodal. For the bimodal, as soon as you have some epsilon of rainfall, half of the area is covered by rainfall. That's normal. These are all the pixels which have a 0 infiltrability are covered by rainfall. These are wet pixels.

I have to say something that I didn't mention precisely is that in our bimodal, we choose probability affected to 0 infiltrability probability equal to 0.5 - half and to the other half also, but we could choose other probabilities, so that means that the crusted parts of the soil occupies half of the area, and the bare soil fraction of the domain occupies the other half. That's normal. As soon as the rainfall rates increase this epsilon, all the pixel with 0 infiltrability area will be covered by runoff. This is what we show here and there's a threshold I don't find where now all the water produced on the 0 infiltrability pixels begin to runon in pixels with the other infiltrability value that is 1 or 2, a big difference.

Mean number of wet area zones, that is the mean number of wet patterns, what we see is the bell curve here. It means that when the rainfall rate is 0, we have almost no wet area zones. These zones are increasing. There are more and more wet patches and then after the rainfall rate which is 0.5, these wet patches begin to connect. There's a coalescence of all the wet areas to form a single one. This is why the mean number tends to 0 when we approach the critical value for the rainfall which is 1, which is the mean infiltrability value.

To prove the importance of the runoff-runon, we showed here on these two figures below. Here, I plot the wet area fraction here as fractional rainfall rates and the rainfall excess area fraction. That is the area which may produce runoff. These were the white areas that I showed you, the 2D example at the top of the transparency.

You see that this rainfall excess area fraction increases when the rainfall rate increases and close to submission, about 40% of the domain is wet due to runon process. When you approach this critical value of the rainfall, the runon process participate to 40% to the wetting of the whole area. I won’t comment on the other figure.

This is what in the geostatistics in connectivity function language we go - the input variable, the structural variable, the dynamical variable, and the functional variable and the connectivity of these two variables are different but what we know is that when the rainfall rates tend to 0, the connectivity of one flowrate variable tends to the connectivity of the infiltration rate variable.

Correlation and nugget effect (ρ<1)

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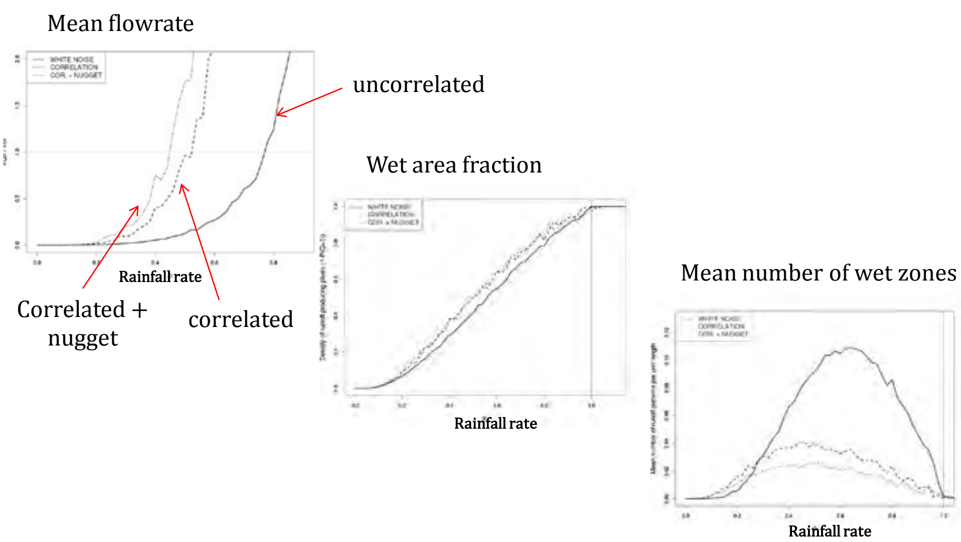


We looked at the effects of the correlation. The introduction of a spatial correlation has huge impact on the mean flowrates. Here is the mean flowrate for the uncorrelated case. That is the log-normal case and here is the index, the mean flowrates for the correlated case. You see that the introduction of correlation increased considerably the mean flowrates. We looked at the introduction of a nugget effect also and in fact, it doesn't change a lot. I won't comment but I just want to say that introduction of nugget doesn't change a lot of things.

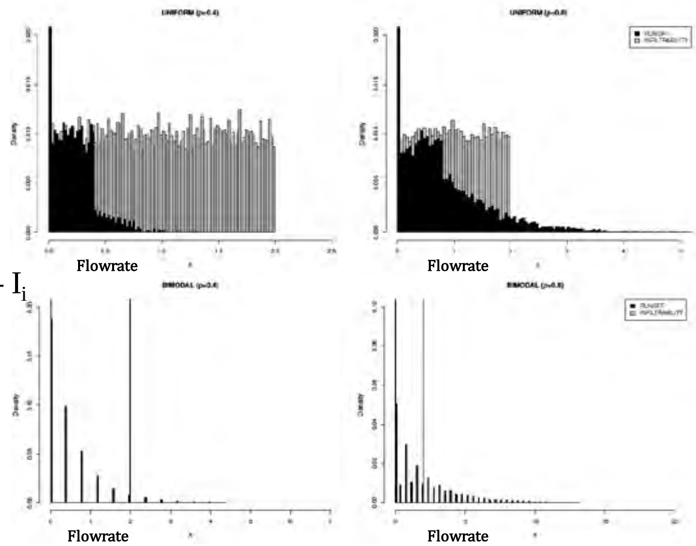
What is interesting is that on the opposite introduction of a correlation does not change the wet area fraction. That's normal because the introduction of a correlation says that - how can you see the introduction of correlation? It means that the area, the size of the – path of the excess rainfall patterns is going to increase but the size of the high infiltrability areas is going also to increase. As a rule, if the size of the wet area patterns increase, the size of the dry area patterns increases also so the wet area fraction does not increase so much.

This is why all the cases - all the wet area fractions are almost the same. I won't comment on the number of wet zones.

PdF of Q (ρ<1)

$Q_i = R - I_i$

$Q_i = Q_{i-1} + R - I_i$

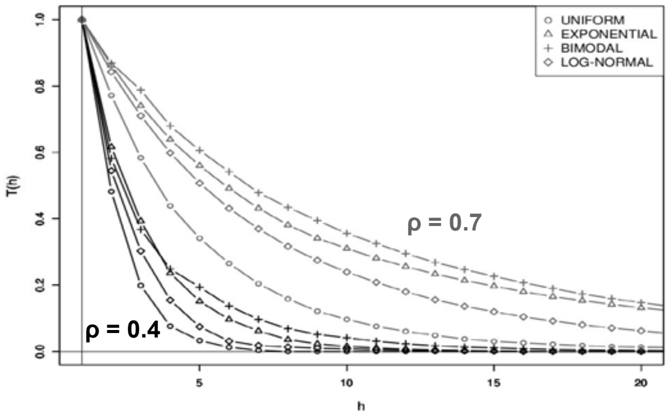


This case, something which is very interesting, I'm going to try to finish soon, is the PdF of Q. We studied the probability function of the values of the flowrate values. Here, I plot what we would take for the uniform case and the bimodal case. Let's consider first the uniform case. For two values of rainfall - I'm sorry but it's very small characters. That's for a rainfall of 0.4 and a rainfall of 0.8.

Here in gray, you have the PdF of the infiltrability. That's uniform from 0 to 2 and the mean is 1. Here's the PdF of the flowrates with the big peak for 0 flowrates which corresponds to the pixels, to the dry pixels. No flowrates. Q is equal to 0. The second population of flowrates displaying - a second population of flowrates comprise between 0 and the rainfall. That is the flowrates occurring on pixel where there is no runon which obeys to this only formulary. You see that I maybe is comprised between 0 and 2. So, it cannot exceed 1 due to the stability criterion. So, it's comprised between 0 and 1, so the area is equal to 1 so Q is lying between 0 and 1. No, excuse me. Here, it's a 0.4. The rainfall is 0.4. The flowrates is lying between 0 and 0.4. This is what we observe and then comes a third population of pixel where you observed runon. That is the pixel where the effective rainfall, the effective inflow flowrate is the rainfall plus what is coming from upward that corresponds to this formula and here you can see that this may be greater than 0.4, maybe 0.8 or whatever. This explained this second population here. This second population, I mean the runoff-runon flowrates increased with the rainfall rates value.

The same for the bimodal. There is something which is very strange for the bimodal is that depending on the value of the rainfall and the value of the probability assigned to each of the modes, you may create different types of flowrate population. Here, there's a single one. Here, you have two flowrate population, one decreasing, another one which is described by some modes, increasing then decreasing. I could show you PdF with four, five, six flowrate distributions. That's a strange mathematical object and we are far to have understood how it works and we are working with Marie Alice at the present time on this. If we have enough results and the findings are enough and clear, we will try to publish it in a mathematical journal like queuing process or I don't know – we see.

Connectivity function (ρ<1)



Hierarchy

Bimodal
V
Exponential
V
Log-Normal
V
Uniform

The connectivity function for this 1D here for the two values of the rainfall and we observed the hierarchy, so the bimodal is far out the distribution which produced the highest runoff flowrates, which produced the largest wet areas and which displays the highest connectivity. The other distribution, the lowest, the connectivity function referential is the uniform which displays a very low connectivity and that is due to the fact that after one pixel of infiltrability, you may expect any infiltrability pixel with an equal probability, and I don't have time here to explain more in detail but as a result, it means that as soon as some runoff, it produced uniform distribution, it has a high probability to infiltrate in the following pixel.

Boundary condition and scale effect

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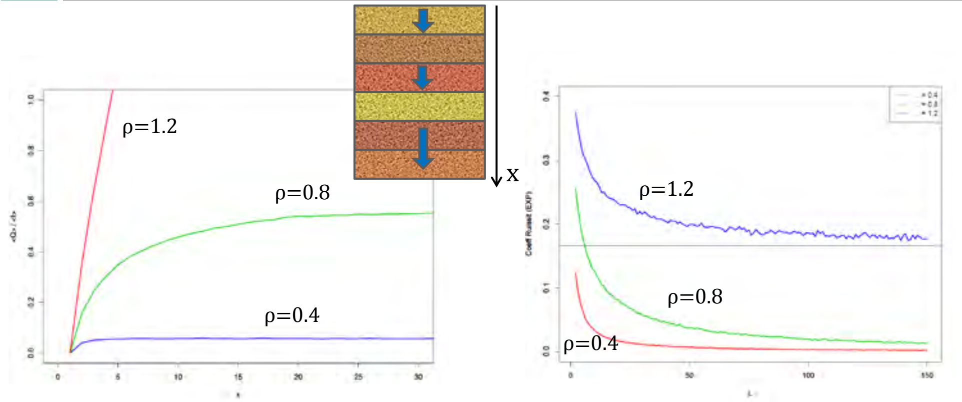

Mean flowrate <Q(x)>   Runoff Coef. Ru(x)

« There is a scale effect on the runoff coefficient.
The highest is the rainfall the more important is the effect »

That’s almost my last transparency before the conclusion.

We studied also the impacts of the boundary condition, then the scale effect. Both are connected so we looked at the impact of boundary connection. That is that we have a boundary connection in our domain which may be the top of the plots or the crest of the hillslope where the inflow flowrate is set to 0. This boundary condition impacts the distribution of flowrates up to a certain distance and we studied this distance. Here you see the mean flowrate as a function of X, X equal to 0 being the location of the highest point of the domain and so you see that this mean flowrate increases up to its stationary value, the values that we were considering just before in the last transparencies.

The other thing that I won’t comment because it would take too much time to comment but the thing that we can remember that we must have in mind is that if I plot the runoff coefficient as a function of the scale observation including the boundary condition – the boundary limits, what we obtain is a runoff coefficient which decreased with the scale of observation and this decrease is more and more slow when the rainfall increases - even when the rainfall has exceeded its critical value of 1, when the runoff overflows all the domain. In fact, when you take into account the boundary condition, there are some probabilities to form some dry areas here close to the boundary condition. This explains why you have this type of decrease.

There's a scale effect of the runoff coefficient. You know that this is a controversial issue. Some people say that due to the runoff-runon process, there's a scale – the runoff-runon process is the source of the scale effects, some others say those are the rainfall distribution. I won't go further into detail.

Conclusion

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- Transient Rainfall : highly complex !
  - > No general equation and need to introduce approximations
- Correlated queues ?
- Topography imposes flow paths
- Link with Erosion

Here are my conclusions. All what I showed you concerns the constant rainfall or the simulated rainfall, let's say. What about the transient rainfall? It's really complex - really, really complex. There's no general equation. I don't know how to put it into a single equation, how to put into a single equation. Runoff dynamics with infiltration on the random surface - with a surface with random infiltrability, but if you introduce some approximation, you can do some things about it and you can use the queuing theory. We are working with Marie Alice on this and we plan to publish a paper in 2013 on this problem.

There's almost nothing known in the queuing process, in the mathematics of queuing process about correlation. We guess that that would be fine to look at this problem with more attention and look at it with the mathematicians. Hydrologists would tell, okay, that's fine, that's very fine, you showed us some mathematical results and so on, but that's not reality. There's a topography which imposes flow paths. The next step also could be how to combine flow paths imposed by topography with random distribution along the flow paths.

By the way, I can say that if you considered not correlated to the distribution, any flow path, along this flow path, the distribution will be white noise but correlated. That's the result so we have to think about how to introduce topography, how to introduce physics in the problem.

I have some friends working in Erosion in this field and they will say that, oh that would be fine to make some connection with the erosion process or at least introduce what you observe in Erosion in your type of model. That’s it. Thank you.

I wish I’ve been clear enough. I'm sorry if there are some points which are not...

＜質疑＞

Male Participant
Thank you very much. That’s a very comprehensive study. One thing that I'm really interested about, you showed that the wet area is actually linearly increasing...

Emmanuel Mouche
...in the rainfall.

Male Participant
Yes, in the rainfall. I thought that if we think about something like rainfall round of response, the wet area might have some exponential or...

Emmanuel Mouche
Yes, it depends on the infiltrability.

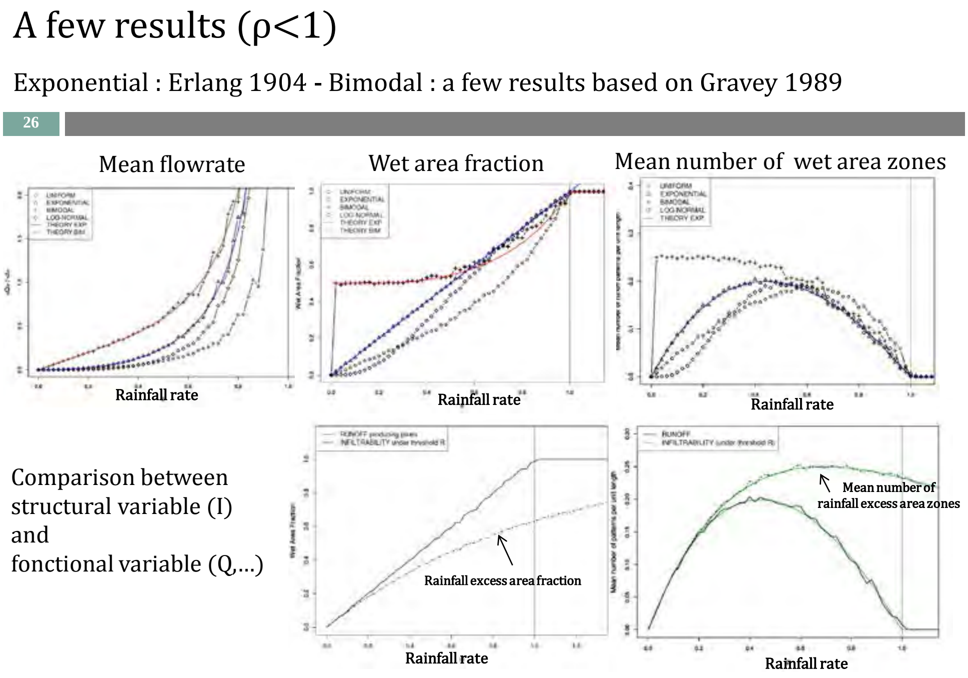
Male Participant
...sometimes nonlinear response but why do you think this kind of linear changing with your results of...

Emmanuel Mouche
Why is it linear? It’s linear in the exponential case. It’s linear...

Male Participant
For instance, at wet area fraction?

Emmanuel Mouche

Yes, it's linear when the infiltrability distribution is exponential. I cannot explain. That's the results of mathematics. Sincerely, I cannot explain physically why it is linear. I don't know. As you said, it's almost nonlinear for the other distribution. What was interesting is that when you consider crusted soils, it becomes highly nonlinear. It reacts differently.



Male Participant

What about like a spatial arrangement of this wet area fractions in these kinds of models?

Emmanuel Mouche

What about – excuse me. I didn't hear you.

Male Participant

Like a spatial– in these models, you are considering some of the - like the pixels, changing the pixels with their eruptions I guess. How this actually – is this like a spatial distributions for the wet areas where the wet areas actually occur? It doesn't make sense.

Emmanuel Mouche
The wet area fraction, the wet area is what the model produces.

Male Participant
Yes and that maybe matters for the linear response.

Emmanuel Mouche
Yes, I'm sorry. Can you reformulate your question?

Male Participant
Okay. If the response - the linear or nonlinear response, maybe it's depending on how we can set up the initial conditions of wet or dry areas, so is this...?

Emmanuel Mouche
Well, it depends primarily of the infiltrability distribution.

Male Participant
Yes, infiltrability distributions in the initial conditions for this model?

Emmanuel Mouche
Yes.

Male Participant
That is if you're changing some initial conditions...

Emmanuel Mouche
You're right.

Male Participant
...you can get the difference.

Emmanuel Mouche
Yes, absolutely. You're right.

Male Participant
It is quite interesting.

Emmanuel Mouche
There are two papers which have been set to what I show you here. One in Double VR in water research and there's another one in ESPL [ph], a special issue of connectivity. We hope that they will be accepted.

Male Participant
Are they coming soon?

Emmanuel Mouche
Next year. Not now. Next year and we are preparing another one where we tackle the transient rainfall case and what we want to show the transient rainfall case with the help of the queuing theory is that you can't have a very strong rainfall, stormy rainfall. If it doesn't last too long, the patches do not have time to connect and you don't produce any runoff flow at the stream. We want to establish some laws between the heights, the importance of the rainfall, the lifetime of the rainfall event, and the runoff prediction or the wet area or the mid flowrates at the stream. This is what we are trying to publish.

Male Participant
Thanks for a very nice presentation. I think it is very complex and smart presentation. As you know, in such hydrology, such relatives are cross-related not only [Unclear] and the land soil process and particularly in the low-lying areas, it cross-relates the [Unclear] processes. The question I think is in these processes, I think hydrology is very cross-related [Unclear] situations…

Emmanuel Mouche
Yes.

Male Participant
…and the usual conditions are also the very important processes about the water in these processes. I think in your queuing theories I think you implicitly increase these effects but I wouldn't know about when you apply

these theories I think the important is whether you have instability status or not. For example, you showed in the final objective in the application during the two dimension processes combining one dimensional [Unclear] processes but in that situation maybe you have some instability situations and the status maybe changed rapidly from one status to another issues. Do you have some experiences like that?

Emmanuel Mouche

I do not see the source of it. Could you precise what you see as instability in the problem? I do not see any source of instabilities with dynamics. For me, no. You are right in the first part of your question is that all this depends of the initial condition of the soil moisture. Many people will argue that the infiltrability is not a good concept that you should use Horton's law at the pixel scale with hydraulic connectivity with the bonding time and so on. Practically, you will never able to be sure that the pixel size when you are in the field. This is what we show in Jérémy. When you are at the field, you can say, hey what's the mean value of the infiltration rates for a given moisture initial state just before the rainfall events. That's all. For us, the infiltrability encompasses different processes which are the states of the initial moisture changes before the rainfall events. The surface of the soil state, I mean the crusted or not and so on because infiltration is not homogeneous vertically and so on. We put it all in a bag and we call it infiltrability; infiltrability for a rainfall event.

Male Participant

I remember, if we assume in the slope, first upstream is the stream slope and changed to [Unclear]. In these situations, I think the average may change greatly. Such models are going to change the [Unclear]. In that situation, I think we are equating just only based on such type of processes.

Emmanuel Mouche

Yes, you mean the connection with the processes like infiltration, we assume that - I mean there's no interaction between runoff. I mean, the runoff going at the surface and any [Unclear] process of the [Unclear] or whatever.

Male Participant

I see. From this point of view, I think it's very interesting. If we apply this theory in that situation, whether infiltrability will occur or not, that has got a question mark.

Emmanuel Mouche
I don't know. I don't know the story.

Male Participant
It's very interesting.

Emmanuel Mouche
No, our modest objective here was to give a framework which helps to understand in very simple situations how the patches, the wet patches which were on the slope connects and arranged to form some flow but some of the domain which maybe plots or a hillslope and so on. You just do some simple physics, because all the papers that we saw are doing a lot of Monte Carlo simulation, very complicated results and there's the final results. They were saying that okay, the mean infiltration rate at a given scale is increasing when the rainfall is decreasing. It does not explain the physics of what happens at the pixel now, how things connect together to form runoff that you observe at the bottom of your plots. This is why this work was in the continuation of Jérémy's work. During Jérémy's work, we observed many things and we said, okay we have to go back to the physics because what you observed is flowrates as a result of the plot scale or the sub catchment but we do not understand. This is why I was interested in your paper because you're intrigued in the explanation of the connection of the patterns. There are more and more papers which are going back to this physics, trying to condition hydrologic models to connectivity - for instance there's a paper of Mueler if you remember which says that you get some good results in the modeling of hydrographs if you introduce in your hydrological model connectivity and runoff and runon process. It's coming now, so that was our aim.

Male Participant
I'm not a specialist about this topic so I'd like to ask a very simple question. What I would like to ask in this slide, row [ph] smaller than 1 means rainfall

intensity is less than infiltration rate.

Emmanuel Mouche
Absolutely, the mean infiltration rate.

Male Participant
The mean flowrate means the slope is the flow rate from this...?

Emmanuel Mouche
That means that you can interpret the mean flowrates either in a statistical way or as a special average of the flowrates.

Male Participant
In the run.

Emmanuel Mouche
Yes, absolutely.

Male Participant
Wet area fraction means percentage of the wet area on the sea.

Emmanuel Mouche
Absolutely.

Male Participant
The other mean number of wet area zone means number of the patches, wet patches?

Emmanuel Mouche
Yes, absolutely.

Male Participant
That means that these wet patches increase due to the rainfall rate.

Emmanuel Mouche
There are more patches.

Male Participant
But all of a sudden, this maybe be recovering.

Emmanuel Mouche
Yes, they coalesce and then decrease and you have a single one patch of infinite [Unclear]. This is when there's overflooding for row is equal to 1.

Male Participant
The same thing you can say that the rainfall system is the same.

Emmanuel Mouche
Absolutely. Yes, you get it.

Male Participant
Thank you very much.

[Japanese]

Female Participant
I don't know my question is precise or not but I have one question. In your experiment, the surface condition is constant or...?

Emmanuel Mouche
The surface?

Female Participant
Yes, for instance very [Unclear].

Emmanuel Mouche
Okay, the rainflow that we consider is the rainflow arriving on the soil. This is the effective rainfall. That's not the rainfall arriving above the canopy, so you may subtract the [Unclear]. If you say that because of the landcover, there's a variability of the intersection, you can't model it through – you can randomize the effective rainfall arriving at the soil because of intersection, I mean randomize. If you have some more intersection here, it means that

you will have on this pixel low rainfall rates. If there are less intersection here, it means that you would have a higher rainfall rates here. You can model it to a random infiltration rate arriving on the soil. Something that I maybe did not understand in your question but I can answer it is that here we do not take into account any evolution with time of rainfall or infiltration. Maybe that was not your question. I'm sorry. Did I answer your question concerning the vegetation?

Female Participant
If the rainfall exists on ground, the root also exists into the soil.

Emmanuel Mouche
Yes, absolutely. This is why we talked about infiltrability. We don't want to talk about Horton process, which is a process that you observed in a sample in lab. We talked about infiltrability which encompassed the impact of the rules and all this process. This is the type of information which is measured on the field. You have a rainfall equal to 1. You have a runoff produced equal to 0.8 so there's 0.2 which is infiltrating and we say that for this rainfall event, we have an infiltration rate of 0.2. That is infiltrability of 0.2. That's the explanation. Well, in fact, it's not as simple as that but that's the spirit, the sense of this modeling.

Female Participant
In your model, high infiltrability means vegetation room might be...

Emmanuel Mouche
For example, yes, absolutely, you're right. Absolutely, yes. It encompass all these processes but in doing that, I do not propose anything. It has been already proposed by Van Dyke, Yu, and Hawkins when the interpreted their scale experiment.

Male Participant
Overall, I'm also interested in scaling issues. I think there's kind of like a two-way approach of scaling. One of the scaling approaches is looking at like your approach looking at hillslope scales and then find out how the node or connectivity developing at the big and catchment scales or maybe sort of

[Unclear] scale. The other approach for these scaling issues is just simply looking at catchments scales and then going back to the details and then find out the node where the model is coming from. I actually did both about the scaling from the hillslope scales to the catchments and the catchments to the scale.

Emmanuel Mouche
There's upscaling and downscaling.

Male Participant
Upscaling and downscaling, yes. For instance, there's some modeling approach and they are quite looking at the catchment scale. They are like small scales, scaling down. Some field observation was of this kind of statistical models and looking at more like small scale approaching to how the statistical functions can explain this kind of connectivity or rows and then finally going up to the rivers and streams and so on.

Emmanuel Mouche
That's an upscaling approach.

Male Participant
How we can actually – it might be a difficult question. How is it emerging this kind of viewing from the hillslope scale to the catchment scale together in this kind of future direction? How we can actually - if you're only looking at hillslope scale, we may be facing some point looking at the catchment scale.

Emmanuel Mouche
Yes, because when you're passing from the hillslope scale to the catchment scale, you have to take into account other processes, there are many runoff. Here's the diffuse runoff. It's not runoff in [Unclear] and there's that point. There's another point which is the exfiltration. The third point is just an experimental point when you measure the stream discharge, what is the runoff part and the underground part of your hydrograph. I don't know. That's very complicated. I wouldn't go so far at the present time. I will just stay on nice system when there's no interaction where the runoff is diffuse.

Male Participant
We can conceptualize some of the processes, but it's always difficult to see some kinds of [Unclear] if we think about the modeling and the processing. I think it's everyone or the hydrologist. I don't know but maybe it become too [Unclear] connection.

Emmanuel Mouche
Yes, you are right.

Male Participant
I think this approach will give us something.

Emmanuel Mouche
We wish to bring some food for thoughts to think about how it happens. In fact, what we want to do here is to put some physics in the understanding of [Unclear] process. This is our aim and say okay, look at the physics, look at the mathematics, may help you to understand some assumptions of how flow is organized. This in France had some good – we had some good [Unclear] about this kind of approach. The models of hydrology in France say, "Hey, go ahead. That seems to be interesting. We're not able to do it, so go ahead." We have some funding from the national research agency perhaps for this project. I think as I told you that the concept of connectivity, that's what you watch in your paper is an emerging interesting concept. That's something very new that the models should incorporate.

Male Participant
[Unclear] method trying to incorporate the physical...

Emmanuel Mouche
Absolutely, yes.

Male Participant
Okay, thank you very much.

[Japanese]

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